Mini-Project 2: These Thumbprints are Oddly Specific:

Skymaps and Payload-Range Diagrams

AE4802 Configuration Aerodynamics

Madeleine Graham

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Problem 1

In our previous courses, we were taught assumptions and simplifications to use while calculating engine thrust. The assumptions were that maximum thrust is reasonable constant with Mach number at sub-sonic speeds and also that the relationship between thrust an altitude can be expressed in some form of the following equation:

$$T = T_0\left(\frac{\rho}{\rho_0}\right) \qquad \text{(Eq 1)}$$

Where T represents thrust, T_o is equal to thrust at sea level, ρ represents air density, and ρ_o is equal to air density at sea level.
Figure 1 shows that in fact the relationship between thrust and Mach number looks more like a curve and that the assumption of linearity is more incorrect at higher altitudes.

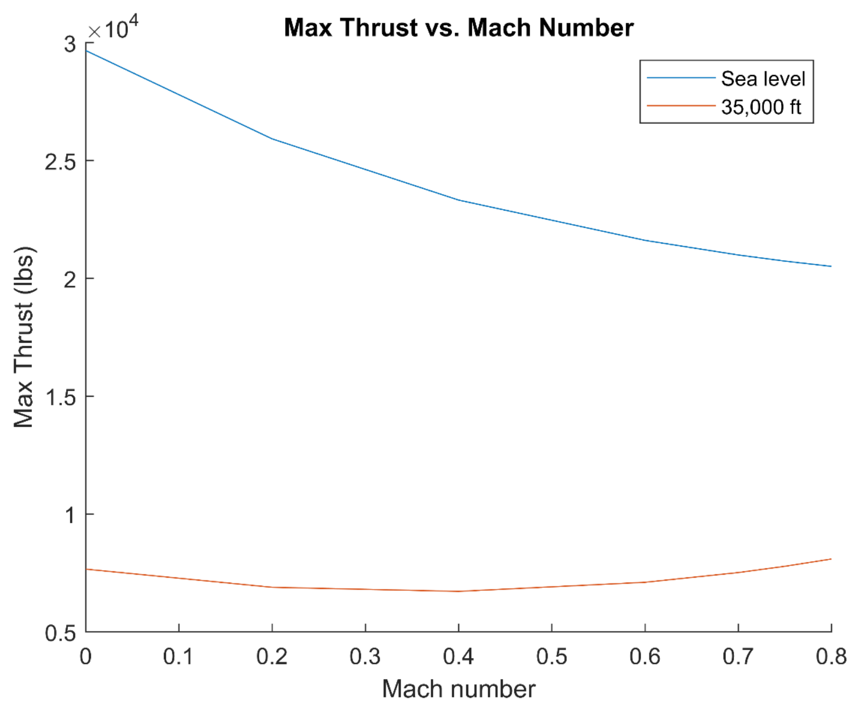

Figure 1: Maximum thrust (lbs) vs. Mach number at sea level and 35,000ft

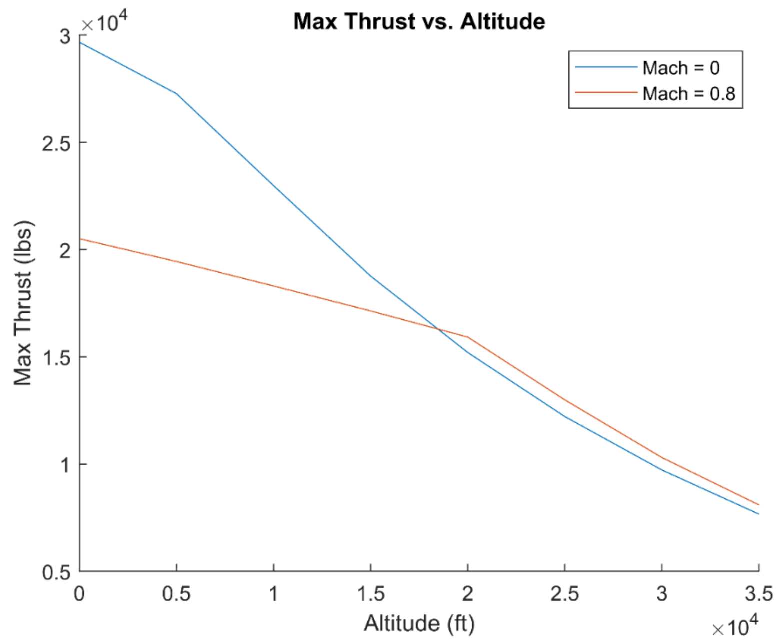

Figure 2: Maximum thrust (lbs) vs. altitude for M = 0 and M = 0.80

Figure 2 shows that the relationship between maximum thrust and altitude is not a simple relationship and cannot be represented by Eq 1. Specifically, at M = 0.8 and at 20,000 ft the curve shows a "notch" that would be impossible to represent in a simple relation. At M = 0 and at a little over 5,000 ft, this "notch" appears again, although less pronounced.

Another assumption made in previous courses was that thrust-specific fuel consumption (TSFC) increases linearly with Mach number but does not vary with altitude. Figure 3 shows the first assumption that TSFC increases linearly with Mach number to be fairly accurate.

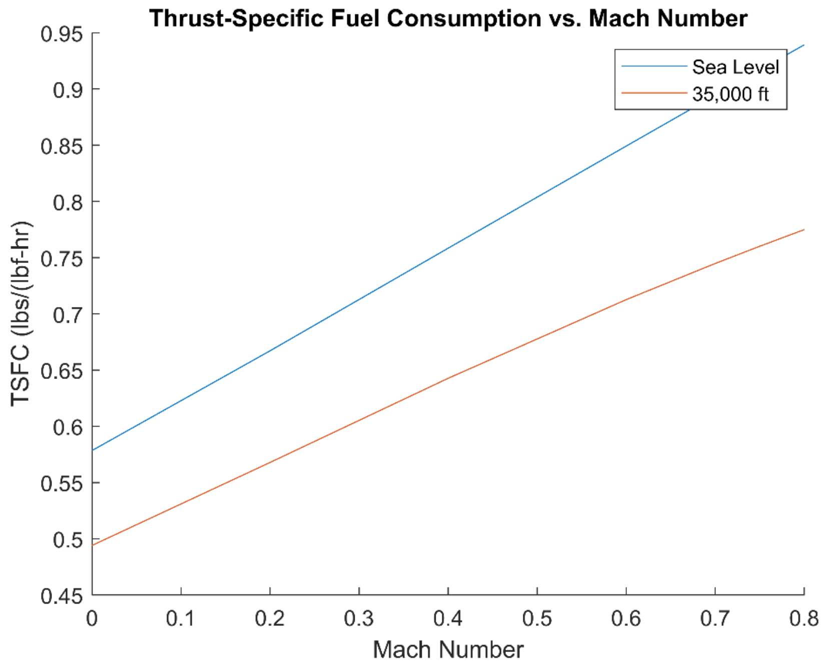

Figure 3: TSFC (lbs/lbs-hr) vs. Mach number at sea level and 35,000 ft

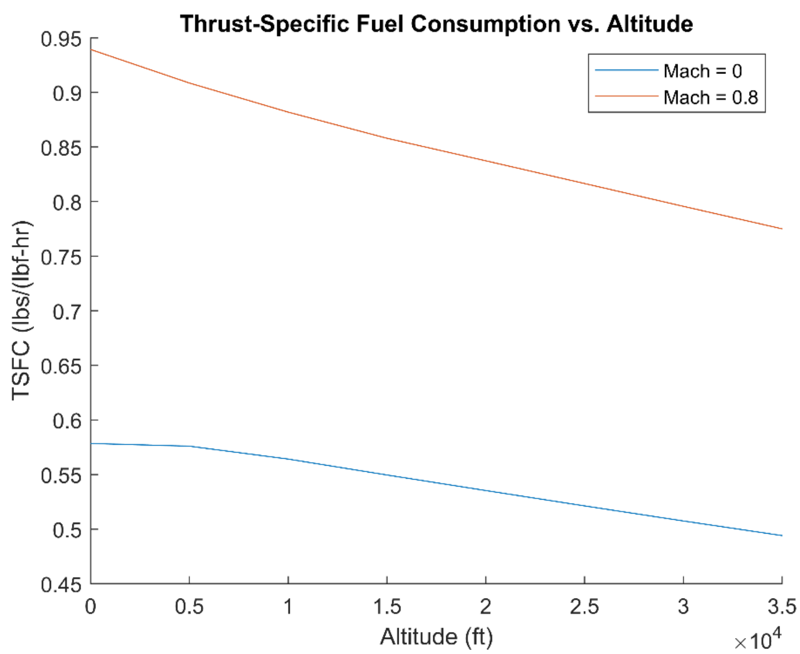

Figure 4: TSFC (lbs/lb-hr) vs altitude at M = 0 and M = 0.80

However, it is also clear from Figure 3 that TSFC does vary with altitude. Figure 4 shows that TSFC plainly decreases with altitude.

I'll give Vehicle Performance a ¾ for its back-of-the-envelope assumptions.

Problem 2

Wave drag only comes into play at transonic speeds because it occurs due to adverse pressure effects that arise when there are local patches of Mach = 1 or greater.

The effect of wave drag is easy to see in Figure 6: Here the values of drag remain constant and depend on C_l. However, when $M = M_{cr}$, suddenly there is a huge spike in drag for each value of C_l. This is the effect of wave drag, as calculated with Lock's 4th Power method:

$$C_{d,w} = \begin{cases} 0; & M < M_{cr} \\ 20(M - M_{cr})^4; & M \geq M_{cr} \end{cases} \quad \text{(Eq 2)}$$

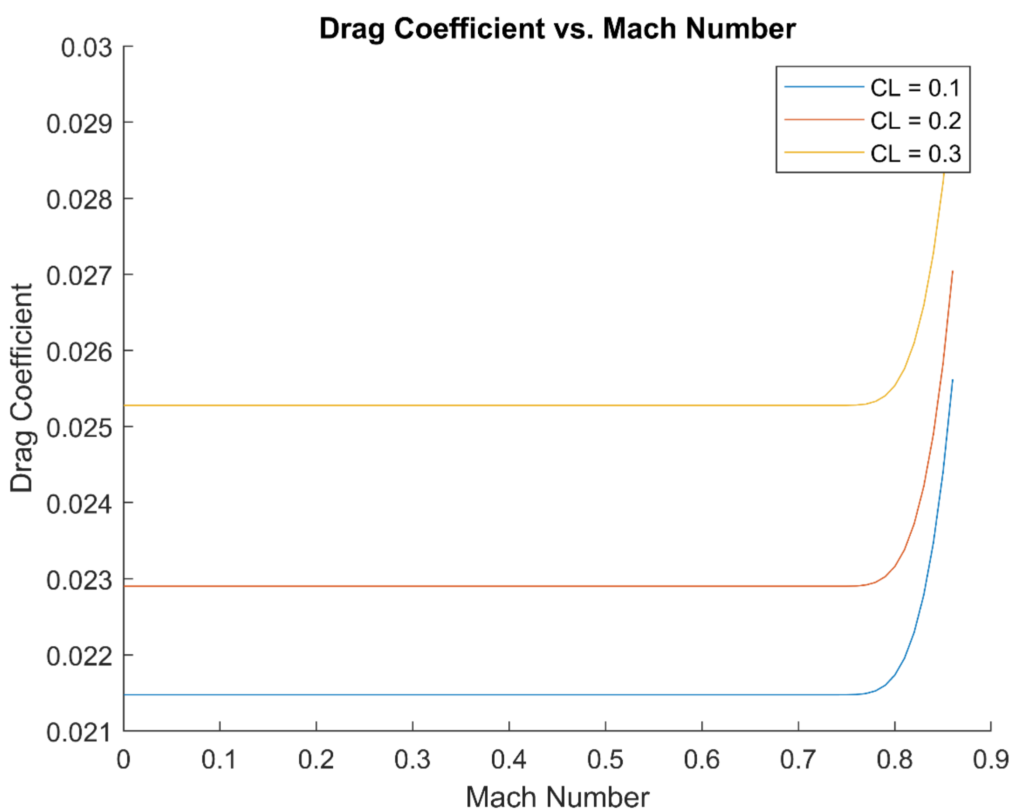

Figure 5: CD vs. Mach number with lines corresponding to CL = 0.1, 0.2, and 0.3

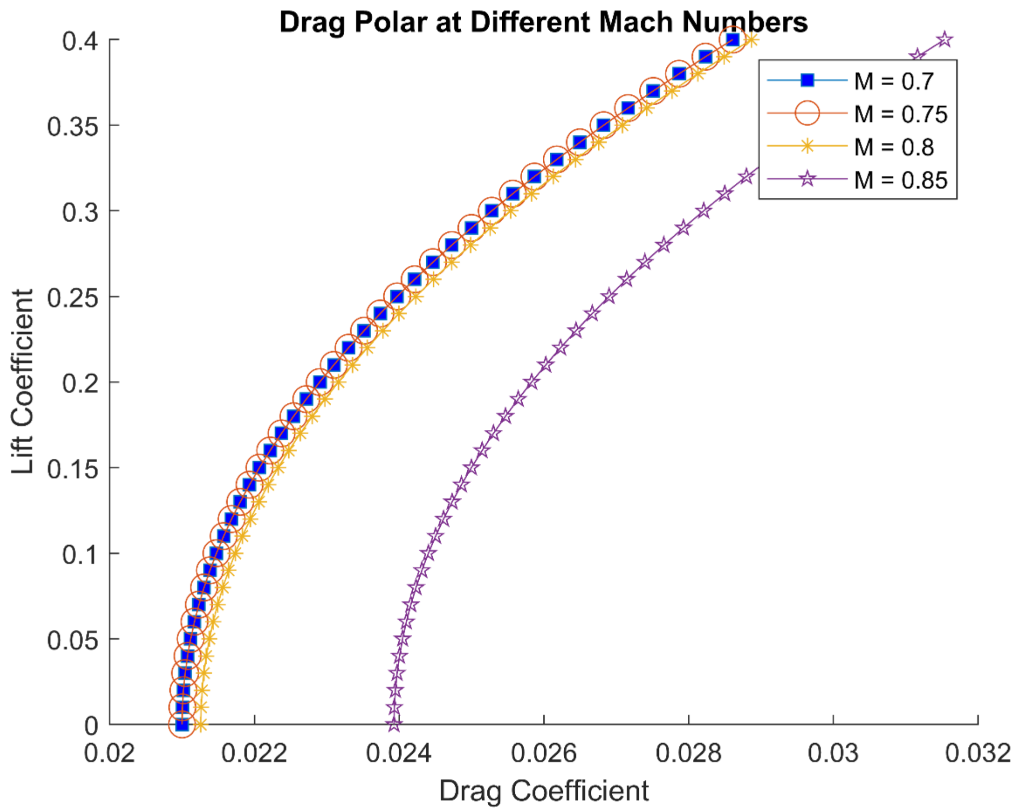

Figure 6: A typical drag polar (Cl vs. Cd) with different curves corresponding to Mach numbers

Figure 6 shows that as wave drag is incorporated using the logic in Lock's 4th Power Method, the drag polar is shifted towards the right. The drag polar curves for Mach numbers that are less than the critical Mach number **are on top of each other**, because they are the same curve.

Problem 3

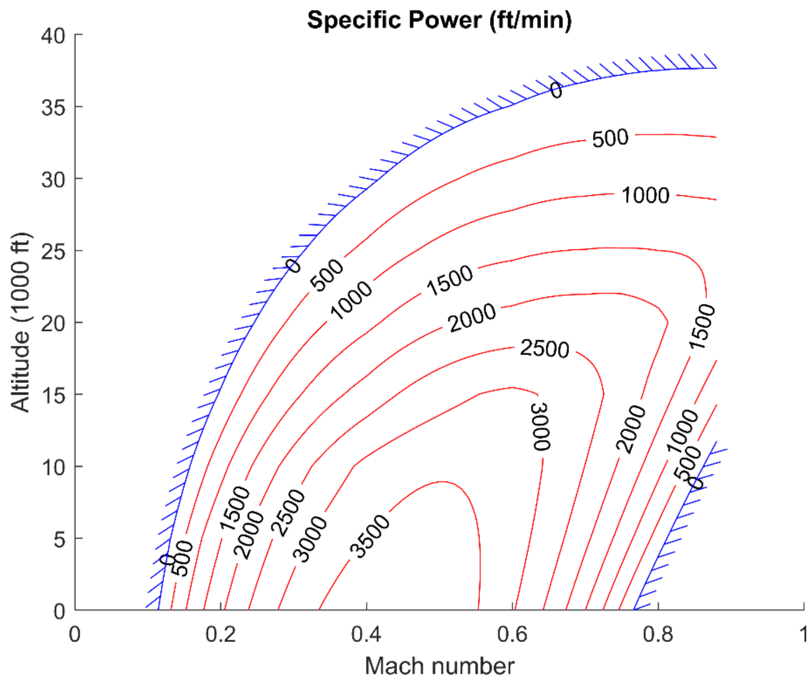

Figure 7: A skymap of specific excess power (ft/min) over the ranges M to 0 to 0.86 and altitude from sea level to 35,000 ft with hatch marks to indicate the boundary of where steady level flight can occur with respect to specific excess power considerations.

Problem 4

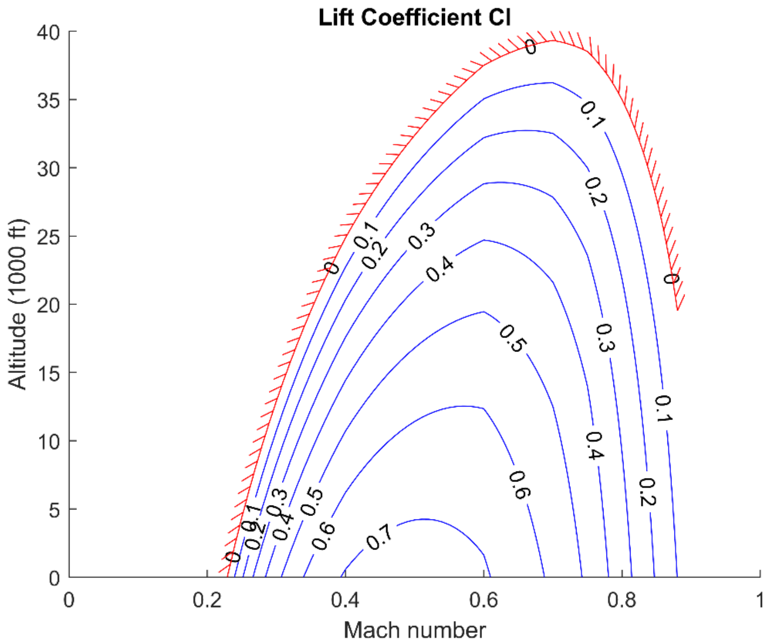

Figure 8: A skymap of lift coefficient over the ranges M to 0 to 0.86 and altitude from sea level to 35,000 ft with hatch marks to indicate the boundary of where steady level flight can occur with respect to buffet Cl considerations.

Problem 5

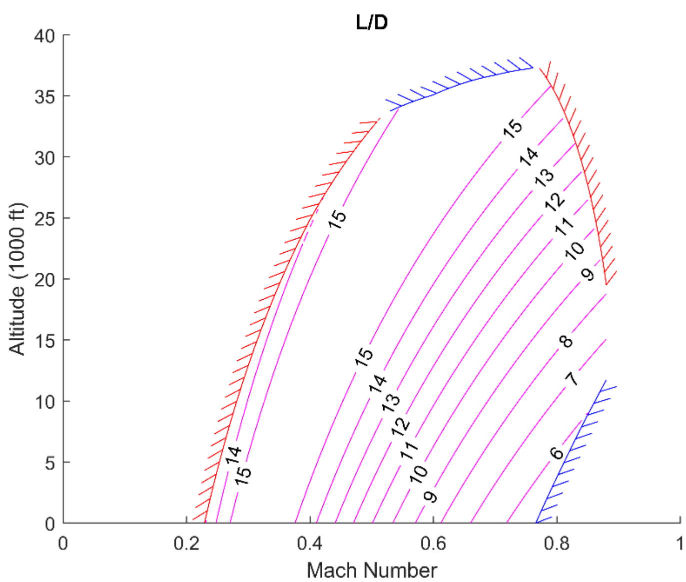

Figure 9: A Skymap of contours of the Lift-To-Drag ratio of our aircraft

For the best Lift-to-drag ratio, I would want to fly somewhere along that plateau in Figure 9, where the contour lines outline a L/D of 15. This is between Mach 0.2 and 0.4 at sea level, but at around 30,000 ft, it is at a Mach number of about 0.5 or 0.6. I would want to fly at those locations because I get the most "bang for my buck" in terms of getting good Lift for all the Drag that it's costing me.

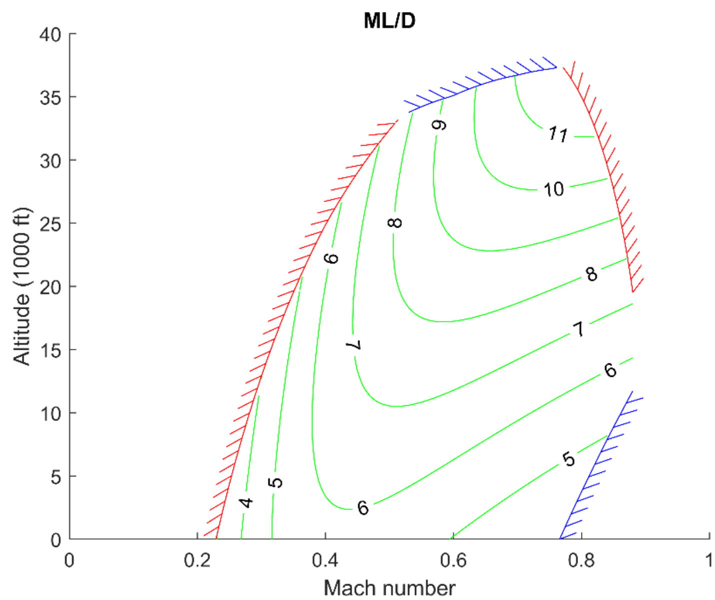

Figure 10: A skymap of contours of Mach number multiplied by the Lift-to-Drag ratio for our aircraft

For the best M* L/D, I would want to fly closest to the tip of the "thumbprint" shown in Figure 10, where the contour line outlines an 11. This is at a Mach number of about 0.7 and at an altitude of around 34,000 ft. I'd want to fly there specifically because I'm getting the fastest speed and best lift for the cost of Drag.

Problem 6

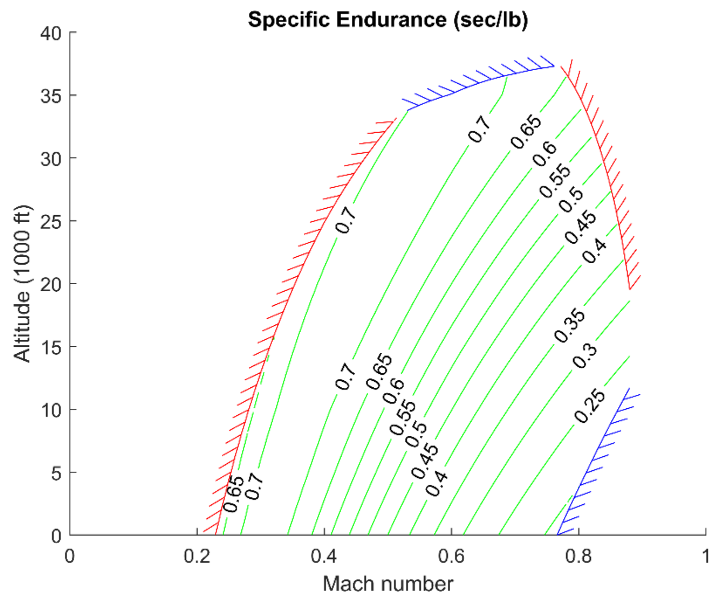

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Figure 11: A skymap with contours of Specific Endurance for our aircraft

I would want to fly along the plateau outlined by the 0.7 sec/lb contour line in Figure 11 to get the best Specific Endurance. That way I can fly for a longer period of time per lb of fuel. This figure bears a resemblance in its contour lines to Figure 9, which makes sense because L/D is one of the drivers of the equation for Specific Endurance (see Eq 3 below).

$$SE = \frac{L/D}{TSFC*W} \quad \text{Eq 3}$$

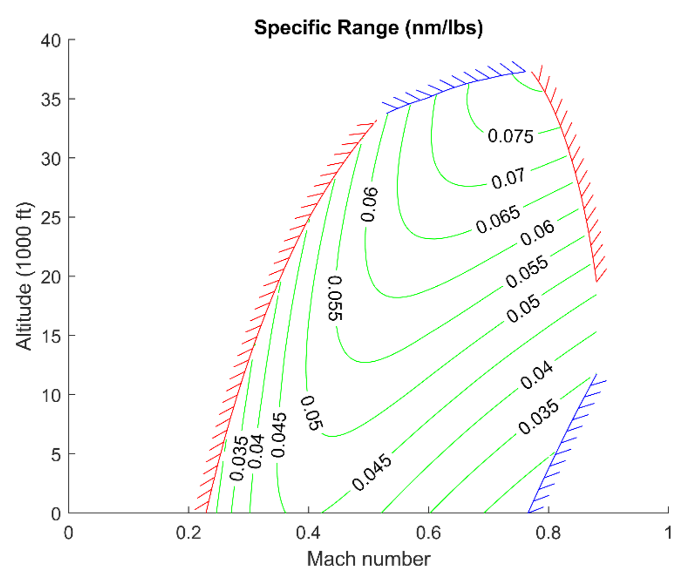

Figure 12: A skymap with contours of Specific Range for our aircraft

To maximize my Range, I'd want to fly at the tip of the "thumbprint" of Figure 12. That way I can fly the farthest per lb of fuel. The skymap for specific range (Figure 12) looks very similar to the contour plot for ML/D (Figure 10) because the equation for SR utilizes M*L/D if you rearrange it to look like Eq 4:

$$SR = \frac{M*L/D}{TSFC*W} \quad \text{Eq 4}$$

Problem 7

Table I: Table of Weights (**in 1000 of lbs**) at Corner Points of a Payload-Range Diagram

Corner Point	1	2	3
Takeoff gross weight	110	110	85
Payload Weight	35	25	0
Zero-fuel Weight	95	85	60
Fuel Weight	15	25	25

(The work to calculate the values in Table I can be found in the attached .m file under Problem 7)

Can the airplane fly with both its maximum payload weight and maximum fuel weight simultaneously?

At corner point 1, the maximum take-off gross weight (MTOGW) of the aircraft intersects with the maximum zero-fuel weight of the aircraft (MZFW). At this point, the payload weight is at its maximum of 35,000 lbs, but the fuel weight is at only 15,000 lbs. Since the MTOGW includes the maximum payload, whatever room there is left is for fuel. Therefore it is **impossible** to have maximum payload and maximum fuel at the same time.

At the other end of the payload-range diagram, as payload weight is reduced, we can start reaching maximum fuel weight.

Problem 8

$$R = \frac{V_\infty (L/D)}{TSFC} \ln\left(\frac{W_i}{W_f}\right) \quad \text{Eq 5}$$

Equation 5 is the Breguet Range equation, which I used to calculate the Ranges for each "corner point" of the payload-range diagram.

At corner point 1, the range of this particular aircraft is **641 nm**. At corner point 2, the range is **1468 nm**, and at corner point 3, the range is **1678 nm**.

The Ranges and weights were calculated from numbers used in Table I. (MATLAB work is in the attached .m file). At each point, I calculated the Lift required for steady-level cruise flight by subtracting the fuel required for take-off (2000 lbs) from the TOGW from Table I. Then I used that information to find all the other parameters (L/D, TSFC, Wi/Wf) in order to calculate range.

Problem 9

Could this aircraft fly from New York to London (around 3000 nm*) with 27,000 lbs of passengers and cargo?

*source https://www.airmilescalculator.com/distance/jfk-to-lhr/

Nope! According to this Payload Range diagram, the trip is utterly outside the capabilities of this aircraft. The flight from JFK to LHR is represented by the red star on the chart at around 3000 nm and with a payload weight of 27,000 lbs, creating an OEW + PW of 87,000 lbs. The ferry range of this aircraft is only around 1600 nm, therefore the trip to London is well outside the capabilities of this aircraft.

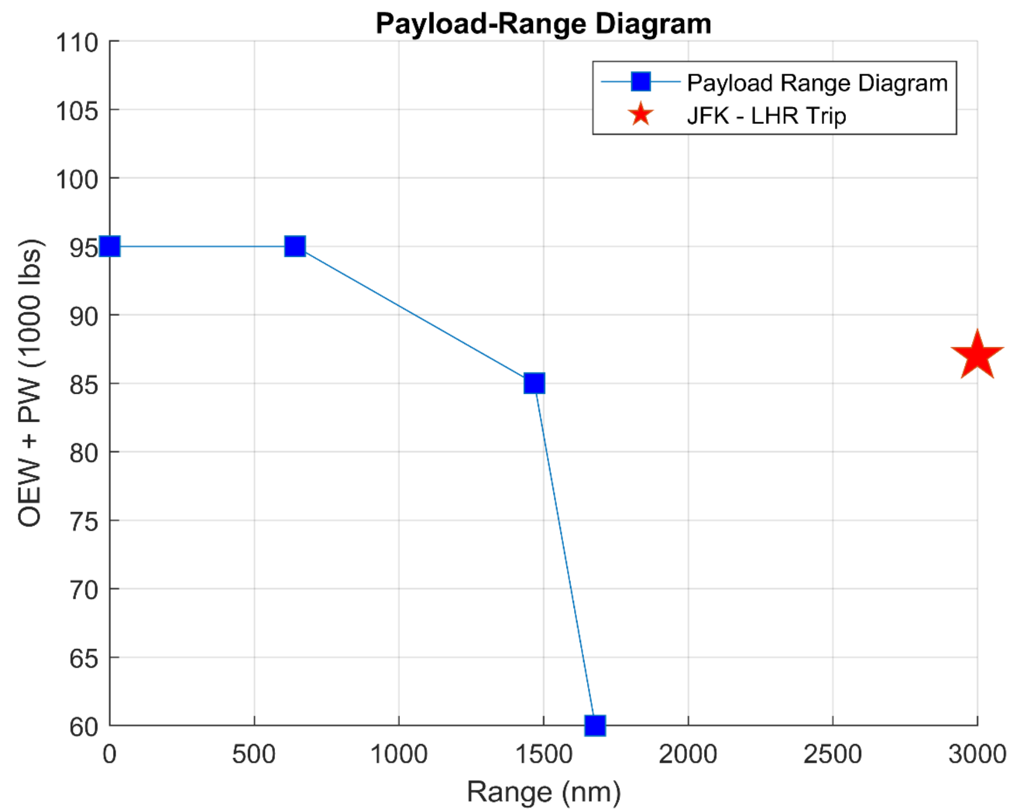

Figure 13: A Payload-range diagram for the aircraft with parameters laid out in this problem